OBSERVATIONAL UPPER LIMITS FOR LOW-DEGREE SOLAR g MODES

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ABSTRACT

A concerted effort to detect global solar g modes using observational data from SOHO and ground-based networks has been carried out. The data from the SOHO SOI/MDI and VIRGO investigations as well as data from the BiSON network have been used. The emphasis has been to look for the dipole modes of low order. Several different approaches have been attempted to enhance the possible g-mode signatures. These methods include: correlation with different time intervals with the same instruments to look for coherent structures; correlation of signals between different instruments to reduce non-coherent solar signals; linear filtering of data from different colours to enhance differences. The detection limit is set by the presence of semi-coherent signal caused by the time development and rotation of solar surface structures. Several peaks have been identified as possible g modes, but unfortunately these results are not consistent either in time or between different reduction methods. We estimate the upper limit of the g-mode amplitudes as 1 mms^{-1} and 0.1 ppm.

1. INTRODUCTION

The objectives of all three helioseismology experiments on SOHO – GOLF, VIRGO and MDI/SOI – include the search for g modes, although it is still not clear whether they exist. After about one year of observations there were no obvious signatures of g modes visible in the power spectra of all three experiments and the methods developed earlier by e.g. Fröhlich and Delache (1984) for detecting g modes failed also. Thus the time for searching in the observations of one single experiment was over and VIRGO initiated a cooperation between the experiments. GOLF was not ready at that time and believed that they would rather continue with their own search. Thus, a g-mode search group was formed from the VIRGO and MDI teams. The objective of this working group, later called the *Phoebus* group as described in Appourchaux (1998), was to join forces not only to improve the way data can be analysed by former and newly developed methods, but also by using combined sets of data to enhance the information content by cross correlations. Thierry Appourchaux of the VIRGO team initiated a first work shop at ESTEC during 3-7 November 1997 (participation: Bo Andersen, Th. Appourchaux, W. Finsterle, C. Fröhlich, D. Gough, J. Provost, P. Scherrer, T. Sekii and Th. Toutain). The results can be found in a report by Appourchaux et al. (1997a). Later the BiSON group was asked to participate in the group's activities and W. Chaplin, Y. Elsworth and G. Isaak joined in December 1997. It became quite clear that the low order p modes would be an excellent test bed to assess the possibilities for g-mode detection (Appourchaux et al. 1998). The ongoing work was discussed during biweekly tele-conferences with lively discussions on how the analysis could be done and about possible g-mode candidates. In the following the first results from this group are reported.

2. DATA USED

The data used in the present analysis start all on 1 May 1996 but are of different length. The power spectra for the range from 100-1000 μHz are shown in Fig.1, 2, 3 and 4. The full line on these plots indicates the limit which one noise peak within 70 μHz could reach or cross with a probability of 10% in the power spectrum; in the following we will call
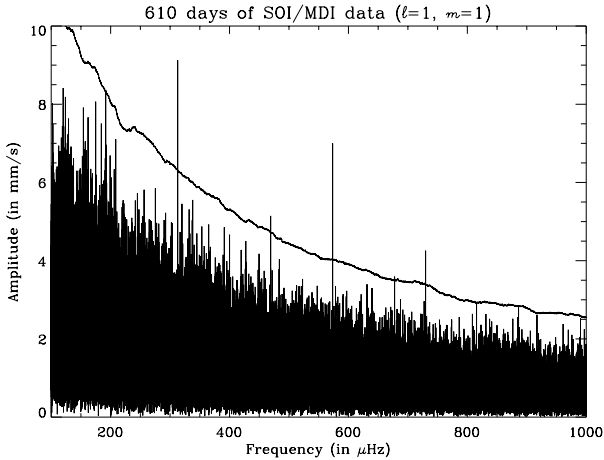
Figure 1. Amplitude spectrum of the 610 days time series of the $l = 1$, $m = 1$ velocity observations of SOI/MDI. The line indicates the 10% level.

Figure 2. Amplitude spectrum of the 733 days time series of the $l = 1$, $m = 1$ intensity observations of LOI. The line indicates the 10% level.
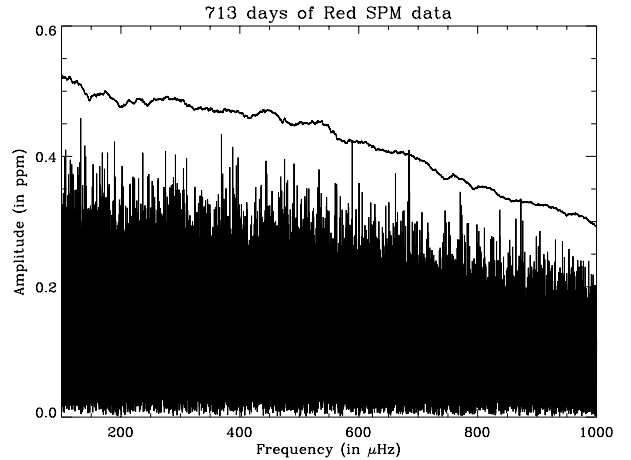

Figure 3. Amplitude spectrum of the 713 days time series of the global sun intensity observations with the red channel of VIRGO/SPM. The line indicates the 10% level.
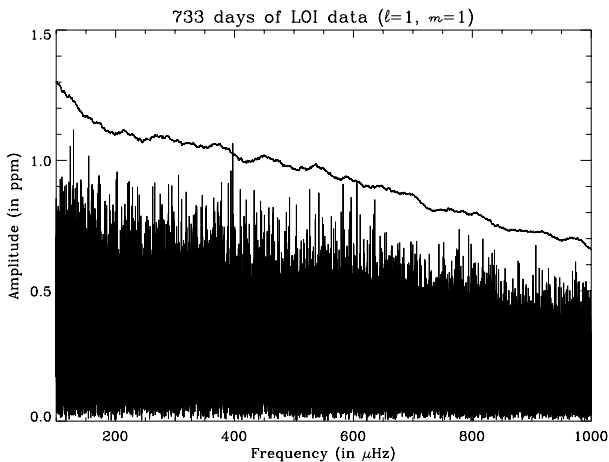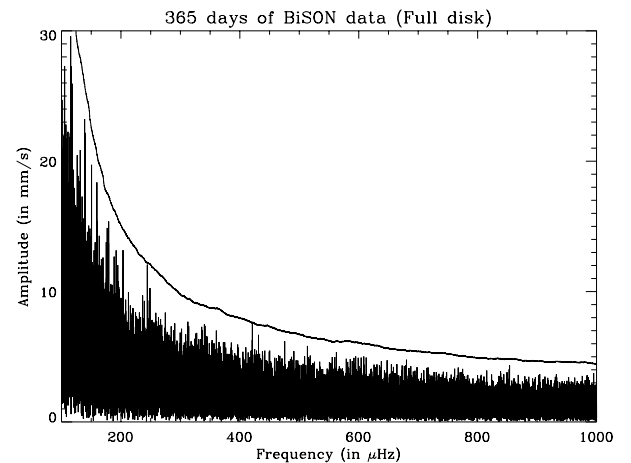

Figure 4. Amplitude spectrum of the 365 days time series of the global sun velocity observations from the BISON network. The line indicates the 10% level.

this limit the 10% limit. The SOI/MDI 610 days of data are the $l = 1$, $m = 1$ velocity signals which are derived from $m - \nu$ diagrams from the 180-pixel LOI-proxy of MDI (Hoeksema et al. 1998). For the calculation of modes the optimal mask technique was used (Toutain et al. 1998). A few peaks are significantly higher than the 10% limit and could be candidates for g modes, but all are harmonics of a spacecraft inherent beating frequency. From VIRGO we have the $l = 1$, $m = 1$ intensity signals from 733 days of LOI (Appourchaux et al. 1997b) analysed as described in Appourchaux and the VIRGO Team (1998) and the full sun intensity signals from 713 days of the PMO6-V radiometer and the three SPM channels (Fröhlich et al. 1997). We see two peaks reaching the 10% limit in the LOI and three peaks in SPM red of Fig. 3. The 365 days of BiSON data are from global sun velocity observations of the 6 station network operated by the Birmingham group (Elsworth et al. 1995; Chaplin et al. 1997). This spectrum is optimized for best signal-to-noise performance at frequencies in the 200-1000 μHz range. At frequencies below 200 μHz, the window function artefacts become important and the spectrum has to be used with care. One peak is

reaching the 10% limit. None of the peaks reaching the 10% limit coincide in all four observations and thus one has to conclude that they are just due to noise.

3. TECHNIQUES TO ISOLATE MODES

The only way to detect g modes is to find methods to distinguish between solar noise and specific signatures from modes. The g modes are supposed to have long lifetimes, thus the signal-to-ratio does improve with time if the noise has a short coherence time, which is usually assumed. From the observation of the solar noise (e.g. Figs.1-4) and by assuming a g-mode amplitude of 1 mms^{-1} we have to wait 20 years before it crosses the 10% limit at 200 μHz (from Eq.2 of Appourchaux (1998)). Thus, we need to develop other techniques to be able to analyse presently existing time series and obviously some additional assumptions have to be made about the g modes and the solar noise in order to proceed.

3.1. Collapsograms

If we assume that for a given degree all tesseral orders are excited one could use this signature for the analysis of resolved observations. This can be used as a simple technique to enhance the amplitude of rotationally split modes by shifting the different m power spectra by the expected splitting and summing them up. The result is called a collapsogram which enhances only a rotationally split mode and not the noise. This technique was successfully tested in the range of low-order p modes and as an example we were able to detect the p_7 mode for $l = 1$ in the SOI/MDI spectrum (Appourchaux et al. 1998). Figure 5 shows as example the $l = 1$ spectra of SOI/MDI velocity in the range where p_1 to p_4 should be found. The 10% level in the lower panel is at 35 power units (this limit cannot be calculated for the upper panel as the statistics is not easily computed for three partly coherent signals added). In the lower panel there is obviously no peak reaching the 10% limit; the high peaks on the other hand are now multiplets, indicating that they are not modes.
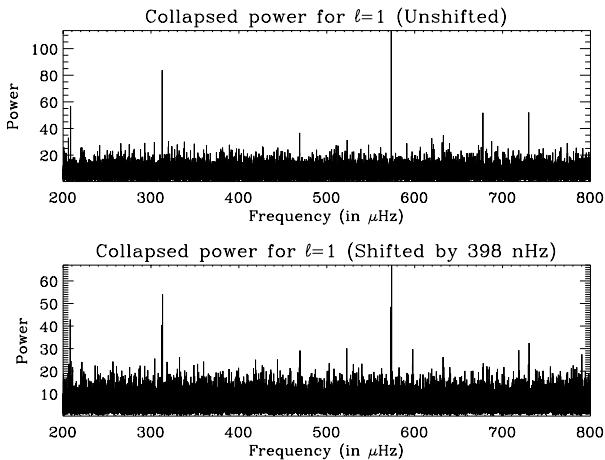

Figure 5.   Collapsed power for $l = 1$ and $m = 0, \pm 1$ not shifted (top) and collapsed power for the same three modes shifted by $m \times 398$ nHz, a reasonable core rotation rate. This is for SOI/MDI velocity in a range where $p_1 - p_4$ would be. The high peaks are artefacts from the spacecraft modulation and the 'multiplet' structure in the lower collapsogram identifies them as such.

3.2. Multivariate Spectral Regression Analysis (MSRA)

The basic idea of this method is to use the information from the red, green and blue SPM channels of VIRGO to explain the solar noise in the total solar irradiance (TSI) measured by PMO6-V. For the three channels transfer functions $B_{r,g,b}^{v}$ are calculated with MSRA (Finsterle and Fröhlich 1998). These describe the gain and phase of the linear filters at each frequency ν which transform the Fourier spectra $f_{r,g,b}^{\nu}$ of the three colors into the one of TSI f_{tsi}^{ν}. Thus the noise reduced power spectrum of TSI becomes

$$F_{tsi}^{\nu} = f_{tsi}^{\nu} - \sum_{j=r,g,b} B_{j}^{\nu} f_{j}^{\nu}. \qquad (1)$$

Before calculating the transfer functions all cross and power spectra have to be smoothed. This smoothing permits to distinguish between incoherent noise and a coherent signal because the coherent signal contributes to the transfer function by only the fraction $1/n$ with n the number of bins over which the spectra are smoothed. In order to test the method a 'g mode' is inserted in all four real spectra with an amplitude of 0.1 ppm in TSI and corresponding amplitudes in the three SPM power spectra (ratios as in the p-mode range). For MSRA the spectra are smoothed over 101 bins or 1.5 μHz. The result shows clearly (bottom panel of Fig. 6) that the 'mode' can be isolated and has now crossed the 10% limit calculated from the reduced spectrum. The coherence squared ρ_{ν}^{2} is a measure for the part of the power spectrum of TSI explained by the three colors. For the example $\rho_{\nu}^{2} = 0.8$, or 80% of the variance is explained. Thus, the reduction of the original noise spectrum of TSI is about a factor 5, which is sufficient to see a 'mode' with 0.1 ppm amplitude; but still not sufficient to see real g modes.
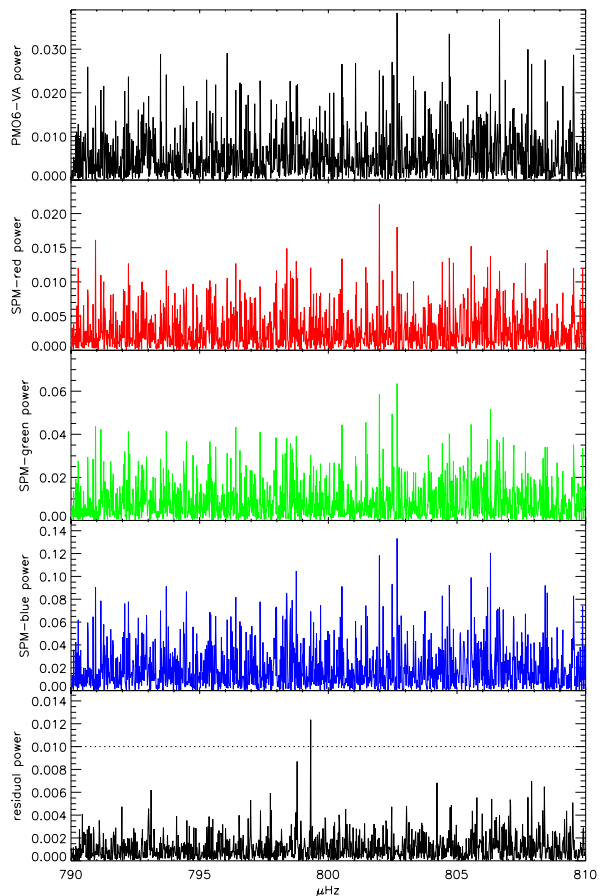

Figure 6.   This plot shows how a hidden 'mode' in the powerspectra can be found with multivariate spectral analysis. The top panels show the four power spectra of TSI and the three SPM channels. After removing the signal explained by the three SPM observations the 'mode' appears as the highest peak (bottom panel), just above the 10% level, indicated by the dotted line.
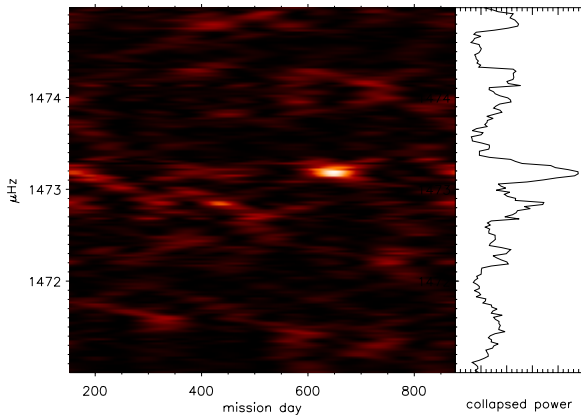
Figure 7. *Amplitude modulation of the p9 as observed by SOI/MDI l = 1 collapsogram. The right panel shows the summed power for each 0.1 µHz bin.*
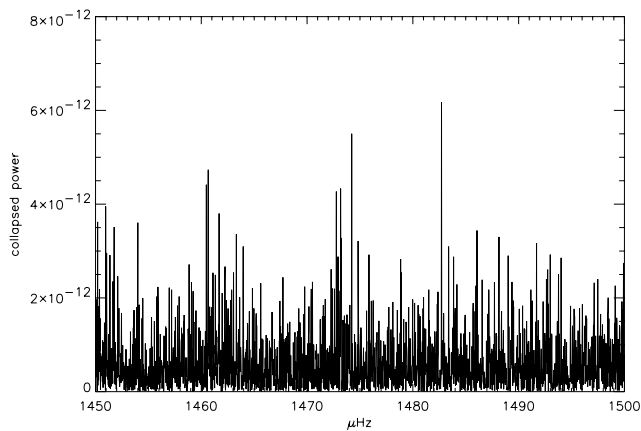

Figure 8. *Collapsed power spectrum for l = 1 from MDI/SOI with the m = ±1 spectra added in the range of p9 at 1473.0 µHz.*

3.3. Amplitude variations

A further technique is to survey the temporal evolution of the amplitudes in selected frequency ranges where modes are expected. Normally this is done with wavelet analysis (e.g. Leifsen et al. 1998b; Leifsen et al. 1998a); here we use the Hilbert transform $\mathbf{H}$ to calculate the envelope of the time series, which is computationally more efficient. $\mathbf{H}$ transforms the time series $X(t)$ by shifting each Fourier component by 90°; thus the amplitude $A(t)$ of the envelope of $X(t)$ is

$$A(t) = \sqrt{X(t)^2 + \mathbf{H}(X(t))^2}. \qquad (2)$$

This can be performed also in a narrow frequency band by calculating the Fourier filtered time series and corresponding $\mathbf{H}$ for the selected range $\Delta\nu$, which determines not only the resolution in the frequency domain as $\Delta\nu$, but also in time as $1/(2\Delta\nu)$. This method is illustrated in Fig.7 for the p mode p_9 as observed by SOI/MDI; the data are from collapsed $l = 1$ spectra and shown for a frequency window centered around the p_9, $l = 1$ frequency. Each frequency band is about 0.1 µHz wide and time re-

Table 1. *Summary of the peaks found with amplitudes above the average power in a 50–100µHz band from a time series of 36 months of BiSON data. The three highest peaks are labelled by *. The column labelled probability gives the estimated probability of seeing the peak within the given range.*

Frequency µHz	Power (cm/s)² per bin	rel. Power	Probability
200.558	0.4621	8.740	0.671
219.631	0.5167	9.772	0.239
231.516	0.4805	9.088	0.474
265.741	0.3717	9.987	0.193
* 288.212	0.4382	11.772	0.032
359.011	0.1936	8.426	0.919
367.653	0.2183	9.499	0.314
382.006	0.2197	9.562	0.295
* 442.338	0.2339	13.626	0.010
502.479	0.1336	10.245	0.298
506.330	0.1192	9.144	0.897
578.368	0.1215	9.316	0.755
654.888	0.1012	9.737	0.495
878.119	0.0839	11.060	0.132
938.892	0.0713	10.290	0.285
* 966.251	0.0834	12.048	0.049

solution is about 50 days. This p mode is hardly discernible in the added power spectrum of $l = 1$ and $m = \pm 1$ of the same time series (Fig. 8), but can be clearly seen as highest peak in the summed power, as shown on the right hand plot. This indicates that the mode is present, but also that the mode is only visible through constructive interaction with solar noise (Appourchaux 1998). As the life-time of this mode is much longer than the time resolution and the high amplitude only present during about 70 days, the observed modulation cannot be due to excitation as most of the variance of p modes in the 5-minute range. This is an important observation which may indicate, that even very low amplitude modes could be seen. The major problem is how to decide whether it is a mode or just noise.

3.4. Cumulative Distribution

The distribution of power of 36 months of BiSON data over 50–100 µHz-wide windows is examined in terms of cumulative distribution of peak power. The power spectrum of a normally distributed noise source should have an exponential probability distribution in the frequency domain. One then expects a fraction $(1 - \exp(-x))$ of peaks in the frequency domain to lie within a relative power x. The spectrum is now searched on the basis of finding high spikes. If one had N independent bins over the interrogated range, then, on average, one would expect to see $N\exp(-x)$ peaks, i.e., this gives an idea of the probability of a peak occurring within this range. There are three peaks (see Table 1) which exceed the 10% limit, and two lie fairly close to the 20th and 33rd daily harmonics (at 265 and 382 µHz). If one examines the g- and p-mode frequency tables in Appourchaux et al. (1997a), the 966 µHz peak lies well away from any of the tabulated, modelled p-mode frequencies. The 442 µHz peak is close to a p_4 peak, but alas with $l = 4$. The 288.2 µHz peak is reasonably close to p_1 $l = 1$ (within roughly 3 to 4 µHz

of the modelled frequencies). It is, however, much closer to g_3, $l = 4$; but again the angular degree is too high, given the spatial filter of the BiSON observing technique. So, none of these peaks seems to be from modes. Moreover, the observed amplitudes of the spikes are higher than expected for low order g or p modes (Toutain et al. 1995). Note that this analysis was performed on a 36 months data set and not on the one the amplitude spectrum is displayed in Fig. 4. This explains the fact that one does not see the three peaks there, and also that the three peaks are noise.

4. CONCLUSION

Several new methods for the search of g-mode have been devised by the *Phoebus* group. None of them has been successful in detecting solar g modes. The amplitude for low order modes must thus be well below 1 mms^{-1} or 0.1 ppm. The interaction with the solar noise may help to amplify the observables, but a unique method to decide whether a peak in time could be due to a mode needs a better understanding of this kind of interaction and of the properties of the solar noise.

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