EFFECTS OF A GAP FILLING METHOD ON P-MODE PARAMETERS

D. Fierry Fraillon and T. Appourchaux
Space Science Department of ESA, ESTEC, 2200 AG Noordwijk, The Netherlands

ABSTRACT

The quality of helioseismological ground based data strongly depends on the presence of gap in the observational window. In order to address that problem in the case of full disk low p-mode velocity measurements, Fossat et al. (1999) proposed a gap filling method called "Repetitive music". The autocorrelation function of the velocity signal shows a correlation of more than 70% at about 4 hours due to the quasi-periodicity of p-mode peaks in the Fourier spectrum. The method then consists in filling gaps of the velocity signal by data, when they exist, located 4 hours before or after.

By using Monte Carlo simulations we assess the effects of the gap filling method on p-mode parameters and their errors. A way to remove the modulation, resulting from the gap-filling method, in the power spectrum is proposed; its effects on p-mode frequencies, linewidths, amplitudes and asymmetries are discussed as a function of both frequency and signal-to-noise ratio of the observational data.

Key words: Sun, p-modes, Gap-filling.

1. INTRODUCTION

The presence of gap in full disk velocity measurements strongly affects the quality of p-mode parameters estimation. For duty cycle lower than 100%, a part of the mode energy is redistributed in sidelobe peaks located at 11.57 μHz around each mode peaks. The superposition between both modes and sidelobes profiles, and the reduction of the signal-to-noise ratio, lead to a p-mode parameters determination with a lower accuracy than the one-hundred-percent duty cycle case. The presence of sidelobes can be taken into account in a fitting procedure (Gelly et al. 1988) in order to avoid systematic bias in p-mode parameters measurements, but the corresponding error bars are still strongly increased by the presence of gap in the velocity signal.

In order to increase the duty cycle of the time series, a gap filling method called "Repetitive Music" (Fossat et al. 1999) is used. Because of the quasi-perodicity of p-mode peaks in the Fourier power spectrum, the p-mode velocity presents a correlation of 70 % at 4 hours (Gabriel et al. 1999). Then the idea of the Repetitive Music is to fill gap in the velocity signal by data, if they exist, located 4 hours before or/and after.

2. THE GAP FILLING METHOD

A way to assess the gap filling effects on the p-mode parameters is to use Monte Carlo simulations. Each realization is generated from the Fourier spectra having a χ^2 with 2 degree-of-freedom distribution and a prescribed mean value (Toutain and Appourchaux 1994, Fierry Fraillon et al. 1998). Simulations of the velocity signal $V(t)$ are then computed using the inverse Fourier transform of the spectra. On each simulation, the gapped temporal set $V_g(t)$ is obtained by applying a window function $W(t)$:

$$V_g(t) = V(t)W(t) \tag{1}$$

The gap filling method is then applied on $V_g(t)$ as follows:

- a gap is filled by the average of data located 4 hours before and after when they both exist
- a gap is filled by data located 4 hours before or after when only one exist
- a gap is not filled when there is no data 4 hours before and after.

The resulting gap filled velocity $V_f(t)$ is given by:

$$\begin{aligned} V_f(t) &= \sum_t^T V_g(t) + \sum_{t_i} V_g(t_i - \tau) + \sum_{t_j} V_g(t_j + \tau) \\ &+ \frac{1}{2} \sum_{t_h} \left(V_g(t_h - \tau) + V_g(t_k + \tau) \right) \end{aligned} \tag{2}$$

Where τ is the filling period ($\tau \simeq 4h$), t=t_i is the case where there is data only four hours before the gap, t=t_j

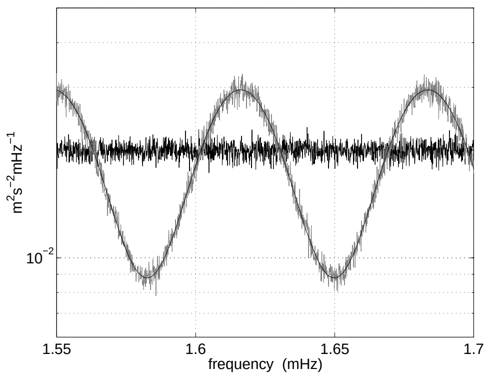

Figure 1. Effects of gap filling on noise. Black: Mean over 500 realizations of an ungapped white noise power spectrum. Gray: Resulting mean power spectrum after application of the gap filling method. Black solid line: analytical model of modulated noise in the case of gap filled spectra computed using the initial value of noise used in the simulation.

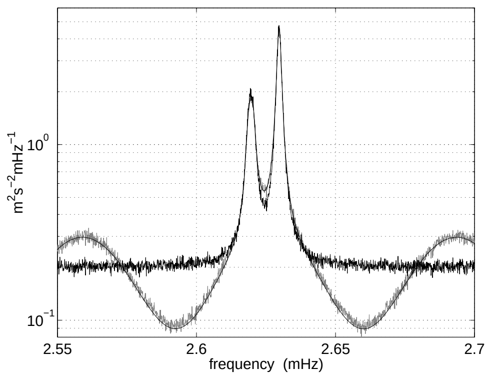

Figure 2. Effects of gap filling on modes, case of $SNR_{l=0}$=20, N=500. *Black: ungapped spectrum for a* l=0-2 *group, computed using an initial set of parameter* $\mathbf{a_0}$. *Gray: corresponding gap filled spectrum. Black solid line: analytical model of modulated mode profile, computed using the same* $\mathbf{a_0}$. *In that case, a* χ^2 *maximum likelihood fit on the gap filled spectrum by the modulated model should produce a correct estimation of the parameters.*

when there is data only four hours after the gap and t=t_h when there is data both before and after the gap.

On each simulated power spectrum, $D(\nu)$=$|\widehat{V}(\nu)|^2$ (ungapped) and $D_f(\nu)$=$|\widehat{V_f}(\nu)|^2$ (gap filled), the p-mode parameters are estimated using a maximum likelihood fit with a model mode profile. Using 500 realizations, the probability density function of each parameter is computed for estimating both its mean value and its error bar. In order to valid the gap filling method, we compute and compare the statistical mean of p-mode parameters coming from the two spectra (ungapped and gap filled).

3. GAP FILLING EFFECTS

The gap filled spectrum $D_f(\nu)$ differs from the original one $D(\nu)$ by a modulation function which depends on the correlation of data in the time serie. The noise has approximately no time correlation, and the mode correlation depends on its lifetime. The modulation is then different for the noise and the mode. But for both case, it is a cos^2-type function derived from Eq. (2). The model of the mode profile $L_m(\mathbf{a},\nu,\tau)$ in gap filled spectra, where **a** is the set of mode parameters, depends only on the gap distribution in the window function and on the value of the filling period τ. This profile is given by:

$$L_m(\mathbf{a},\nu,\tau) = L_0(\mathbf{a},\nu)M_l(\nu,\tau) + bM_b(\nu,\tau) \qquad (3)$$

Where $L_0(\mathbf{a},\nu)$ is the usual mode profile (Lorentz or asymmetric profile) and b is a noise constant value. The noise modulation $M_b(\nu,\tau)$ and the mode modulation $M_l(\nu,\tau)$ are both derived from Eq. (2) and does not depend on the mode parameters.

Figures 1, 3 and 2 shows, for three different signal-to-noise ratio, the effects of the gap filling method on a mean l=0-2 power spectrum, computed using an initial set of parameters $\mathbf{a_0}$, and the gap filled mode model $L_m(\mathbf{a_0},\nu,\tau)$ computed using the same set $\mathbf{a_0}$. The noise modulation $M_b(\nu,\tau)$ recovers the simulated one (Fig. 1).

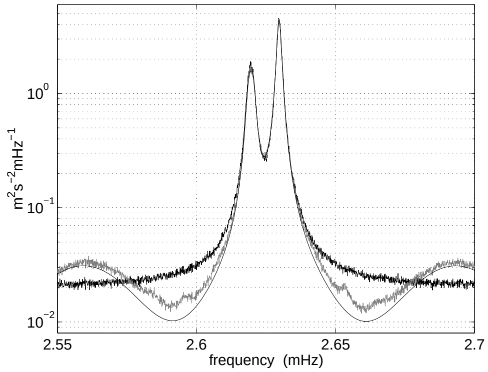

Figure 3. Effects of gap filling on modes, case of $SNR_{l=0}$=200, N=500 *realizations. Black: mean ungapped spectrum for a* l=0-2 *group, computed using an initial set of parameter* $\mathbf{a_0}$. *Gray: corresponding gap filled spectrum. Black solid line: analytical mode profile for gap filled spectra computed using the same* $\mathbf{a_0}$. *In that case, the p-mode parameter estimation by using the analytical gap filled profile might produce some bias which may mainly affect parameter like asymmetry and noise.*

The analytical mode modulation $M_l(\nu,\tau)$ has been approximated for low signal-to-noise ratio. We have neglected some terms in the analytic expression of $M_l(\nu,\tau)$ because theses terms are minimum close to the mode frequency and tend to 0 when the signal-to-noise ratio is decreasing. Then $L_m(\mathbf{a_0},\nu,\tau)$ recovers the simulated gap filled spectrum for low signal-to-noise ratio (Fig. 2) while it does not totally for higher SNR (Fig. 3). In addition, as shown hereafter, neglecting some terms in $M_l(\nu,\tau)$ does not lead to a measurable bias on p-mode parameters.

4. RECOVERING THE CORRECT VALUE ?

The data used are simulated velocity in full disk measurement with a duration of observation of 2 months. The signal-to-noise ratio varies from 1 to 200. Only $l = 0$, 1 and 2 p modes from 1.5 mHz to 4 mHz are present. The window function is an original IRIS window function (David Salabert private communication) modified in order to obtain a one-hundred-percent duty cycle after gap filling. When the gap filling method is applied with the original IRIS window function the duty cycle reachs from 55% to 96%. For reaching a 100 % duty cycle the window function has been modify by substituting a 0 by a 1 on 4% of the time serie length, it corresponds to the presence of gap without data located 4 hours before and after.

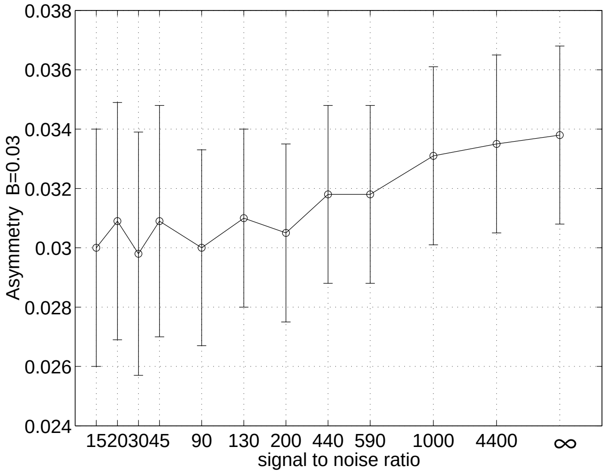

Figure 4. Asymmetry recovery as a function of signal-to-noise ratio. This parameter is estimated using 500 realizations of an asymmetric l=0 mode computed for several signal-to-noise ratio. The initial value is B=0.03, this is the most sensible parameter to the SNR. Even if the mode profile $L_m(\nu, \tau)$ is an approximation for high signal-to-noise ratio, the asymmetry estimated on gap filled spectra is correct for $SNR < 600$: the bias is well within the error bars for a single realization.

Monte-Carlo simulations of ungapped and gap filled spectra are used for estimating the parameters and their error bars. The gap filled spectrum is first computed using one single value of τ which is chosen in order to center the modulation on mode group at 3 mHz. For low (ν<2 mHz) and high (ν>3 mHz) frequencies, the gap filled spectrum is also computed using the value of τ corresponding to the mode group studied ($\Delta\tau/\tau$=±0.04%). The precise value of the filling period τ is determined by placing the maximum of the modulation on the mean frequency of a mode group ($\cos^2(\pi\nu\tau)$=1 for which $\nu = (\nu_{l=0} + \nu_{l=2})/2$).

The p-mode parameters are estimated by a maximum likelihood fit of the appropriate mode profile, using the two degree-of-freedom χ^2 distribution of both spectra. The goal is to make the comparison between the statistical mean value of two parameters coming from two different spectra in order to see if the gap filling method gives correct parameters values.

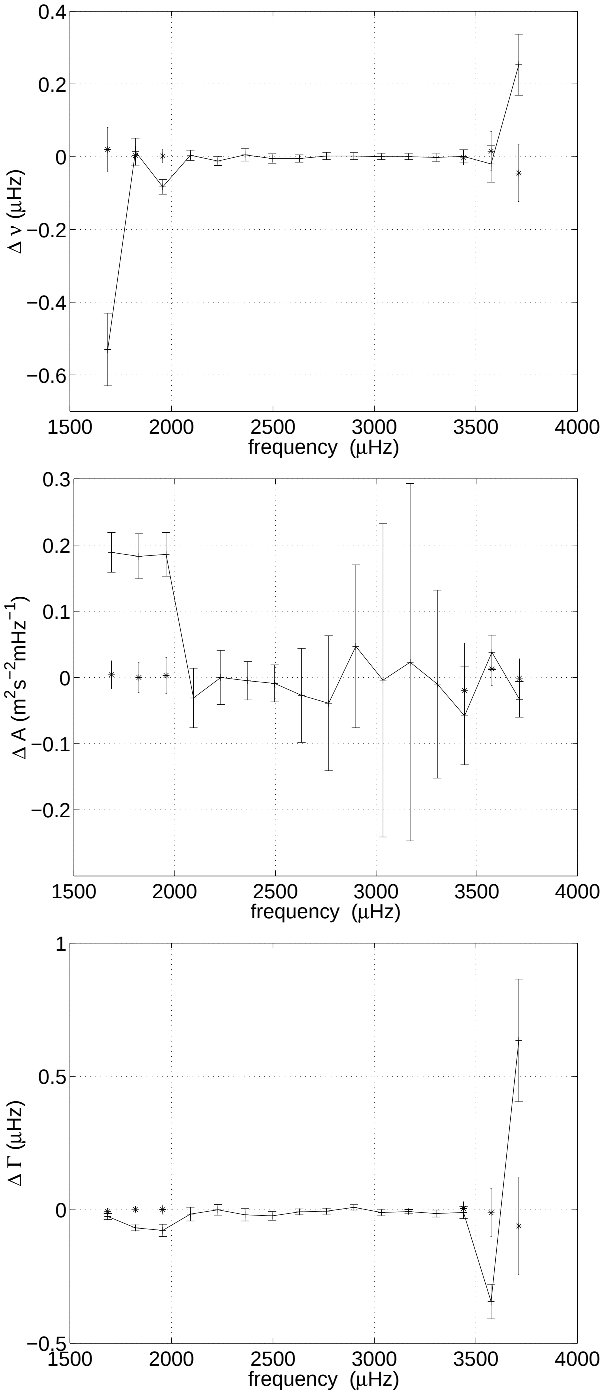

Figure 5. Differences between parameters estimated on gap filled spectrum and on ungapped spectrum (taken as the reference), l=0. Crosses : differences using one value of the filling period τ. Stars : Same differences using the appropriate value of τ. The error bars are statistical error bars divided by the square root of N (N=500), then the systematic bias due to the gap filling method is well within statistic variations, and therefore negligible.

Figure 4 shows the recovery of asymmetry as a function of signal-to-noise ratio. The recovery of this parameter is sensible to the approximation of the modulation function

$M_l(\nu, \tau)$ as expected from figures 2 and 3.

Figure 5 shows the recovery of amplitude, frequency and linewidth as a function of frequency for a l=0 mode. The recovery of theses parameters depends on the filling period τ used and it is not affect by variation of linewidth and signal-to-noise ratio. The results for l=1 and 2 are similar.

5. CONCLUSION

An analytical mode profile has been tested for gap filled spectra obtained by the "Repetitive Music" method (Fossat et al. 1999). This modulated profile depends on the distribution of gap in time series and can be estimated only with the knowledge of the temporal window function.

We show that fitting gap filled spectra using this analytical model allows to recover the correct p-mode parameters, when the signal-to-noise ratio is lower than 600 and when the correct value of the filling period τ is used. The value of τ is settled by centering the $\cos^2(\pi\nu\tau)$-type function on the mean frequency of the mode group (l=0-2 or l=1-3) studied.

This gap-filling method can be used for any solar seismic data (either ground- or space-based) but also for any stellar seismic data. For example, stellar data acquired at one site have typically 16-hour data gap. For such gaps, the ideal candidate would be a star with a large separation corresponding to 68 μHz giving maximum temporal correlation at 8 hours (i.e. respectively 136 μHz and 4 hours in the solar case). This would put the duty cycle to 100%. Unfortunately, this 'best' scheme will produce a modulated spectrum that will look like what is looked for, e.g. regularly-spaced peaks. This gap-filled method is in any case rather suited when the data are partially replicated: two stations is better than one.

The interest of the method is then to increase the duty cycle of any gapped asteroseismic data without losing information on p-mode parameters. The signal-to-noise ratio is increased and the presence of sidelobes is limited leading to a p-mode parameters estimation with a better accuracy, and negligible systematic bias. This gap filling method, and the derived mode profile, can be applied in the p-modes frequency range on all kind of full disk data. It concerns mainly ground based experiment looking at the Sun (BISON, IRIS, LOWL-integrated, GONG-integrated) or Procyon (ELODIE) but also space data (ACRIM).

REFERENCES

Fierry Fraillon D., Gelly B., Schmider F.X., Hill F., Fossat E., and Pantel A., 1998, Astron. Astrophys. 333, 362

Fossat E., Kholikov Sh., Gelly B., Schmider F.X., Fierry Fraillon D., Grec G., Pallé P., Cacciani A., Ehgamberdiev S., Hoeksema J.T., and Lazrek M., 1999, Astron. Astrophys. 343, 608

Gabriel M., Grec G., Renaud C., 1999, Astron. Astrophys. 338, 1109

Toutain T. and Appourchaux T., 1994, Astron. Astrophys. 289, 649