On expressing mode splitting with Clebsh-Gordan coefficients and related issues

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September 14, 2001

1 Goal

The goal of this document is to express the splittings given in terms of $(\nu_m - \nu_0)/m$ as a function of the a_i expansion based on Clebsh-Gordan coefficients.

2 Background

In the antique age of helioseismology (before the 90's or so), mode splittings were usually expressed in term of Legendre polynomials. Unfortunately, these polynomials are orthogonal only on a continuous space (between [-1,1]) not on a discrete set such as (-1,0,+1) for l=1, for exanple. Therefore, other expansion are required that can either be computed by hand or derived from quantum mechanics.

3 Clebsh-Gordan expansion

Riztwoller and Lavely (1991) derived the following polynomials from quantum mechanics. The splittings are expressed as follows:

$$\nu_{(l,m)} - \nu_{(l,0)} = \sum_{i=1}^{i=n} a_i l \mathcal{P}_l^i(m) \tag{1}$$

where

$$\mathcal{P}_l^1(m) = \frac{m}{l} \tag{2}$$

$$\mathcal{P}_l^2(m) = \frac{6m^2 - 2l(l+1)}{6l^2 - 2l(l+1)} \tag{3}$$

$$\mathcal{P}_l^3(m) = \frac{20m^3 - 4m(3l(l+1)-1)}{20l^3 - 4l(3l(l+1)-1)} \tag{4}$$

$$\mathcal{P}_l^4(m) = \frac{70m^4 - 10m^2(6l(l+1)-5) + 6l(l+1)(l(l+1)-2)}{70l^4 - 10l^2(6l(l+1)-5) + 6l(l+1)(l(l+1)-2)} \tag{5}$$

Please note that for all i we have $\mathcal{P}_l^i(l) = 1$. The polynomials are derived from Eqs (39) to (44) of Riztwoller and Lavely (1991). The derivation of $(\nu_m - \nu_0)/m$ is then straightforward using Eqs (1) to (5).
Bon courage!