Peak bagging for solar-like stars

Thierry Appourchaux
Research and Science Support Department, European Space Agency, Keplerlaan 1, 2200AG, Noordwijk, The Netherlands

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Abstract. The identification of the low-degree p modes in other stars is the challenge of future asteroseismology space missions such as COROT, MONS, MOST or Eddington. The identification is based on a priori knowledge of the characteristics of the modes. We shall review the most common assumptions needed for the identification such as basic stellar structure, visibilities, rotational splittings or linewidths. We shall describe a few tools needed for facilitating the identification. As soon as modes are properly identified, the peakbagging of the mode characteristics can be done using Maximum Likelihood Estimation. We give examples of the whole process using solar data and hare-and-hound exercises performed in the frame work of the COROT project.

Keywords: Sun, stars, seismology

1. Introduction

In the very near future, there shall be a fleet of space missions aiming at understanding the internal structure of the stars: MOST[1], COROT[2], MONS[3] and Eddington[4]. All these missions will observe global oscillations of the stars by measuring tiny light fluctuations; they are due to the perturbation of the star surface by the oscillations. The detection and identification of these modes of oscillation is the challenge of all these missions. This challenge is for most stars extremely difficult (e.g. Cepheids) but easier for solar-like stars. For these latter, the Sun has been and is a great aid and example.

Hereafter after having defined what is meant by solar-like stars, I explain how the current identification can be based on that of the

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[1] Microvariability and Oscillations of Stars, a Canadian mission to be launched in April 2003 (Matthews et al., 2000)
[2] COnvection and ROTation, a CNES mission to be launched in Nov 2005 (Baglin and The COROT Team, 1998)
[3] Measuring Oscillations in Nearby Stars, a Danish mission under study to be launched in 2005 (Kjeldsen et al., 2000)
[4] A mission part of the 'Cosmic Vision' programme of ESA, to be launched in 2008 (Favata et al., 2000)

Sun case. When the identification is achieved, peak bagging[5] can be performed; the theory of which is explained here. The practice of mode identification and peak bagging is developed through the use of hare-and-hound exercise. The results of such exercises carried out by the COROT Data Reduction Group are presented here.

2. What is a solar-like star?

The definition of a solar-like star depends on whom you talk to. There are basically 3 'definitions' driven by:

1. *the stellar structure*: the solar-like star is similar to the Sun, i.e. with an outer convection zone and a radiative zone. Stars with a mass smaller than 1.2 $M_\odot$ satisfy the criterion as they do not have a convective core yet (Iben and Ehrman, 1962; Cox and Giuli, 1968).

2. *the excitation process of the oscillations*: the process is similar to that of the Sun, i.e. due to turbulent convection (Houdek et al., 1999). It occurs in stars with mass smaller than 2 $M_\odot$ with an outer convection zone even if it is very shallow (Houdek et al., 1999); the efficiency of the process is driven by the turbulent Mach number M_t (Houdek et al., 1999) that is maximum for 1.6 $M_\odot$. Above roughly 1.5 $M_\odot$ the stars have overstable modes; they enter the Cepheids instability strip (Houdek et al., 1999).

3. *the structure of the oscillation spectrum*: a regular spacing of the modes must be observed as predicted by asymptotic frequencies (Tassoul, 1980). The regular spacing provides the basis for a diagnostic tool: the *echelle diagramme* devised by Grec (1981) for the Sun.

I prefer the last definition because the structure of the ridges in the echelle diagramme of a solar-like star looks like those of the Sun. The ridge structure can simply be derived from the asymtotic frequency expression given by Tassoul (1980):

$$\nu_{n,l} \approx (n + \frac{l}{2} + \epsilon)\Delta\nu_0 - \frac{Al(l+1)}{n + \frac{l}{2} + \epsilon} \tag{1}$$

where n is the order of the modes, l is the degree, $\Delta\nu_0$ is the large frequency separation (or acoustic diameter of the star); and A and ϵ

[5] peak bagging is a term coined by Jesper Schou who is a keen moutain climber. Such individuals use to record climbed peaks in a book and referred to it as the peak bagging list...

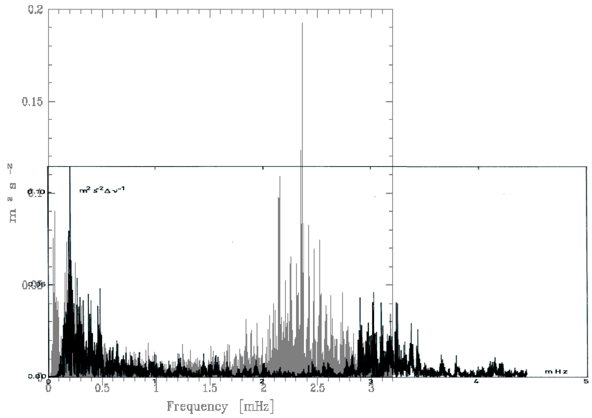

Figure 1. The power spectra of the Sun observed by Grec and Fossat (1977) (Black) and of α Cen observed by Bouchy and Carrier (2002) (Grey). Thanks to modern technology, they were put to the same scale showing that the stellar observations of today match the solar observations of the 70's. Both spectra show what can be obtained for a few days of observation stellar radial velocities. The frequency at which the modes are maximum depends upon the mass of the star (Houdek et al, 1999).

are constant involving integrals over the stellar model. From Eq (1), it can be noted that the even modes ($l = 0-2$) and odd modes ($l = 1-3$) have nearly the same frequency; they are only separated by the small frequency separation, i.e. δ_{02} or δ_{13} related to the second term in Eq (1). In the solar case, $\delta_{02} = 10\mu$Hz for $l = 0-2$ and $\delta_{13} = 13\mu$Hz for $l = 1-3$. That is this difference that allowed helioseimologists to identify properly the degree of the ridges. It outlines that the experience gained in helioseismology will be of considerable help for mode identification.

3. Past and present mode-identification methodology

3.1. In the early age of helioseismology

The first detection and identification of the solar oscillations as global modes (low degree) is attributed to Claverie et al. (1979). They identified the regular spacing based on the prediction made for low degree, low order mode frequencies by Iben and Mahaffy (1976). They had enough foresight for finding that the detected modes were higher order modes of about $n > 20$. Unfortunately, they could not identify the degree of the modes because the length of the time series did not allow for separating the odd degrees ($l = 1-3$) from the even degree ($l = 0-2$) (See above). Such an identification was performed by Grec et al. (1980) using the echelle diagramme and the properties of the small frequency separation. The length of the time series was such that they had to collapse the power along the ridges in order to increase the signal-to-noise ratio (Fossat et al., 1981). At this stage, such a technique allowed to resolve the rotational splitting (Grec et al., 1983). The identification of the order n was made by identifying the f-mode ridge in the (l, ν) diagramme obtained by making images of the Sun (Deubner, 1975).

3.2. In the XXIst century and beyond

From helioseismology, we can identify three steps in the mode identification, all using the echelle diagramme:

1. determination of the large frequency separation (i.e. $\Delta\nu_0$)
2. degree identification and determination of the small frequency separation (i.e. δ_{02}, δ_{13})
3. rotational splitting and star inclination estimation

These steps are intimately linked to the length of the time series and the signal-to-noise ratio. A typical large frequency separation for a solar-like star ranges from 40 μHz to 170 μHz which is easily resolved by observing a star over a few hours (Audard and Provost, 1994). The small frequency separation is more difficult to retrieve as it ranges typically from 3 to 13 μHz (Audard and Provost, 1994), and requires a few days before resolving it. The visibility of the various degrees can also be used for identifying the modes in the second step. For instance the $l = 3$ modes are significantly damped by the integration over the stellar disk (Christensen-Dalsgaard and Gough, 1982 for velocity and Toutain and Gouttebroze, 1993 for intensity); it renders the $(l = 0 - 2)$ ridges significantly different from those of the $(l = 1 - 3)$ ridges in the solar case. The rotational splitting is even more challenging: it does require a few months of observation in the case of the Sun, as its rotation period is of the order of a month.

The order identification requires more technological developments. In the not too distant future space based interferometers may provide the first low-resolution images of the stars. If it were possible to make higher-resolution images of the stars, the f-mode identification would certainly be achieved for stars; this is likely to a be dream for the XXIst century and beyond.

At the time of writing, only two stars have been observed showing unambiguously solar-like p modes: the Sun and α Cen. These two stars are the test bed of the identification process described above. Figure 1 shows what can be obtained by observing the Sun (Grec and Fossat, 1977) and α Cen (Bouchy and Carrier, 2002) over a few days. The large and small frequency separation are easily identified for both stars. As for the splitting, the time series are still too short for α Cen for revealing the splitting. Careful attempts have been made by (Bouchy and Carrier, 2002) for deriving the rotational splitting of the modes of α Cen but more observing time is required. The measurements of the rotational splitting for low-degree modes is rather difficult. That is only in the mid-90's, that reliable splittings could be obtained for the Sun using the theory of power spectrum fitting. It is likely that such measurements for the stellar case will require extreme care before getting some confidence in the results, but as we show hereafter the theory of power spectrum fitting is now well mastered.

4. Theory of peak bagging

After tagging each ridge in the echelle diagramme with a degree l, we proceed with fitting the power spectrum or performing peak bagging. The theory of power spectrum fitting for the Sun as a star is now well developed. It has been described in numerous articles and is now well understood; it is based on the use of Maximum Likelihood Estimators (MLE) (See Duvall and Harvey, 1986 and Appourchaux et al., 1998 for a review). The power spectrum is fitted using MLE that is by assuming:

- a statistics for the spectrum
- a model for the modes

The statistics of the spectrum for uninterrupted time series is a χ^2 with 2 degree of freedom (Gabriel, 1994). The model of the modes includes the mode frequency, the mode linewidth, the mode amplitude, the mode profile and the background noise. In some case, modes of different degrees overlaps ($l = 0 - 2$, $l = 1 - 3$), they are usually fitted together. The simultaneous fitting of these modes requires to know the visibility of the modes. For intensity this is given by Toutain and Gouttebroze (1993) while for velocity this is given by Christensen-Dalsgaard and Gough (1982). The rotational mode splitting and the inclination of the star have to be taken into account. There are up to $2l + 1$ components depending on the inclination of the star. The star inclination is taken into account depending on the way the modes are observed (velocity or intensity). The spherical harmonics eigenfunctions has to be weighted by the projection onto the line of sight or by the limb darkening. For intensity, the visibilities of the modes can simply be approximated using the decomposition given by quantum mechanics rotation matrices (Toutain and Gouttebroze, 1993).

Error bars on the parameters are derived using the inverse of the Hessian (or curvature matrix) (Appourchaux et al., 1998; Toutain and Appourchaux, 1994). The error bars derived give a good estimate of the true error bars (Appourchaux et al., 1998). The validity of such error bars can be tested using the z test as described by Chaplin et al. (1998); this is a test comparing internal and external error bars.

Finally, the significance of the fitted parameters (under the H_0 hypothesis) can be checked using the likelihood ratio test as described by Appourchaux et al. (1998). This test is rather useful for assessing the adequacy of the model fitted to the data.

5. Mode identification and peak bagging in practice: hare-and-hound exercise

As coined by an anonymous scientist:'In theory there is no difference between theory and practice; in practice there is'. The theoretical approach described above needs to be tested with *real* data. In the history of helioseismology, the *real* data have been sometimes fabricated in a such a way that it looked like a road runner chase. The term hare and hound[6] appeared in the GONG[7] Newsletter #9 of 1988 when the GONG Inversion team performed simulated inversion on data fabricated by a hare (Douglas Gough).

Within the COROT team, we found that it would be useful to have such a hare-and-hound exercise that would simulate (as well as stimulate) the mode identification and the peak bagging. The steps for this exercise are the following:

- a Team A generates theoretical mode frequencies and synthetic time series
- a Team B analyzes the time series, performs mode identification, peak fitting and structure inversion

The two teams have no access to any other information but the time series and the known characteristics of the star. Nothing else is allowed.

5.1. Team A: The making of synthetic time series

The steps for making the artificial time series are very similar to those needed for using MLE. You need to assume:

[6] In French we would rather call it: *le jeu du chat et de la souris*
[7] Global Oscillation Network Group

- a statistics for the (Fourier) spectrum
- a model for the modes

The statistics of each component of the Fourier spectrum is assumed to be normally distributed with a zero mean and an rms value described by the model for the modes. For the model of the modes, the following data are needed:

- the characteristics of the solar-like star and the theoretical mode frequencies (or asymptotic mode frequencies (à la Tassoul))
- the mode visibility (Christensen-Dalsgaard and Gough, 1982; Toutain and Gouttebroze, 1993)
- the excitation profile and linewidth (Houdek et al., 1999; Samadi and Goupil, 2001; Samadi et al., 2001)
- the rotational splitting and stellar inclination
- the background stellar noise derived from that of Trampedach et al. (1998) or of the Sun as in Harvey (1993)
- any other parameter (asymmetry, trick, mind twister or anything else to make it more *real*)

As soon as each Fourier component is properly modeled, an inverse Fourier transform provides the necessary time series that will be passed to the next team.

5.2. Team B: Power spectrum analysis

The power spectrum analysis can be done in various ways. Here I recommend to perform the steps as described in Section 3 and 4, i.e. to make an echelle diagramme of the power spectrum, perform mode identification and then to fit the modes according to a model. When the p-mode parameters are obtained the last task of this team is to invert the mode frequencies in order to derive the stellar structure. This is beyond the scope of this talk but this is discussed by Berthomieu et al in these proceedings in the frame work of the COROT HH exercise described hereafter.

5.3. The COROT HH exercises

The Asteroseismology and Exoplanet-search mission of the French space agency (CNES) is going to be launched at the end of 2005. There are several scientific groups being involved in the preparation of the mission. The Seismology Working Group (SWG) prepares the data analysis and scientific interpretation of the seismology data. The Data reduction group of the SWG is more precisely in charge of the data analysis aspects. This group has set up three different hare-and-hound exercises[8]:

- HH#1 Validation of power spectrum fitting technique (no inversion, asymptotic frequencies)
- HH#2 Recovery of the initial stellar model for synthetic stars (full cycle as described above)
- HH#3 Choice of targets

The first exercise is over and lead to the validation of power spectrum fitting performed by different groups (Institut d'Astrophysique Spatiale, Orsay: Boumier; Observatoire de Nice: Toutain; European Space Agency: Appourchaux). The last exercise is on going. The second HH exercise is over and I report on some of the results obtained.

5.3.1. *The teams*

There were three teams involved in the process:

- *Meudon*: Observatoire de Meudon (stellar model and inversion) + Appourchaux (time series and power spectrum fitting)
- *Nice*: Observatoire de Nice (stellar model and inversion) + Toutain (time series and power spectrum fitting)
- *Queen Mary*: Queen Mary stellar (model and inversion) + Barban (time series and power spectrum fitting)

Each team produced synthetic time series and passed it on to the 2 other groups for power spectrum fitting and structure inversion.

5.3.2. *A piece-of-cake case: Nice synthetic data*

Figure 2 shows the echelle diagramme obtained for the *Nice* synthetic data. The time series are 150-day long and sampled at 60 sec. The frequencies fitted are obtained within about 0.1 to 0.2 μHz of the theoretical frequencies; the splittings and the star inclination are also recovered. Similar results were obtained with an other piece-of-cake time series that of the *Meudon* team. The frequency comparisons and inversion associated with this time series are presented elsewhere in these proceedings by Berthomieu et al.

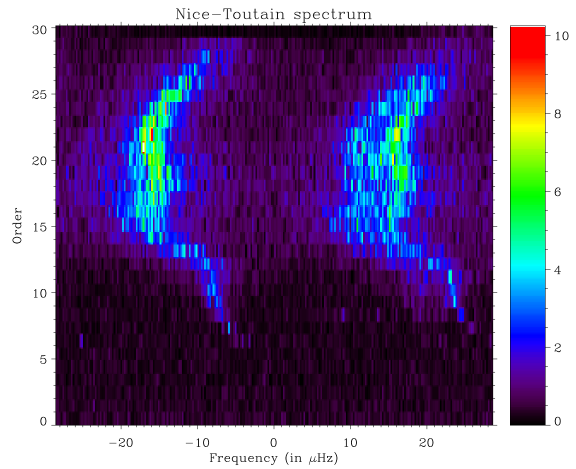

Figure 2. Echelle diagramme of the *Nice* time series. The large separation is about 58 μHz. The ridge at the left hand side is due to the $l = 1$ modes. The identification is rather easy because the ridge at the right hand side show a more complex structure related to the splitting of the $l = 2$ modes interfering with the $l = 0$ mode ridge. The order labeling the y axis have no absolute meaning.

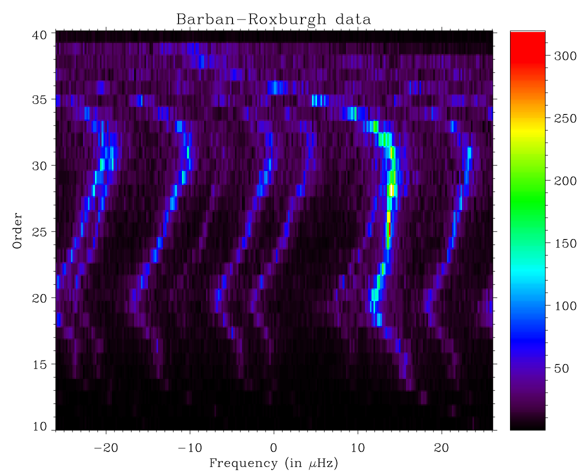

Figure 3. Echelle diagramme of the *Queen-Mary* time series. The large separation is about 52 μHz. The bright ridge at +12 μHz is identified as due to the $l = 0$ modes; it is the only one ridge with this shape. The 3 parallel ridges at -23μHz, -13μHz and -3μHz are attributed to the $m = -1$, $m = 0$ and $m = +1$ modes of $l = 1$; the splitting is about 10μHz. The ridges left over (5 of them) were attributed to the $l = 2$ modes; the $m = 0$ mode ridge crosses the $l = 0$ mode ridge. The $m = -2$ and $m = -1$ mode ridges seemed to be on either side of the $l = 1, m = +1$ mode ridge, while the $m = +1$ and $m = +2$ mode ridge are on either side of the $l = 1, m = -1$ ridge.

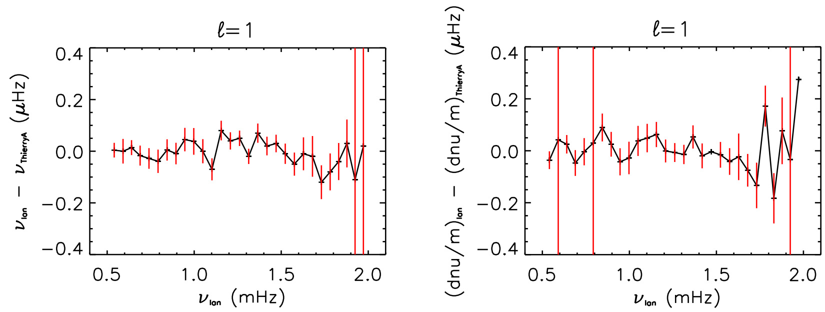

Figure 4. Comparison between the parameters fitted by the *Meudon* team with the theoretical parameters of the *Queen Mary* team; (left) for the $l = 1$ mode frequencies, (right) for the $l = 1$ mode splitting. The agreement between the input and the fit is excellent for $l = 1$; a similar agreement was obtained for $l = 0$. The error bars on the the freqeuncies are typically ranging from 0.05 μHz to 0.1 μHz.

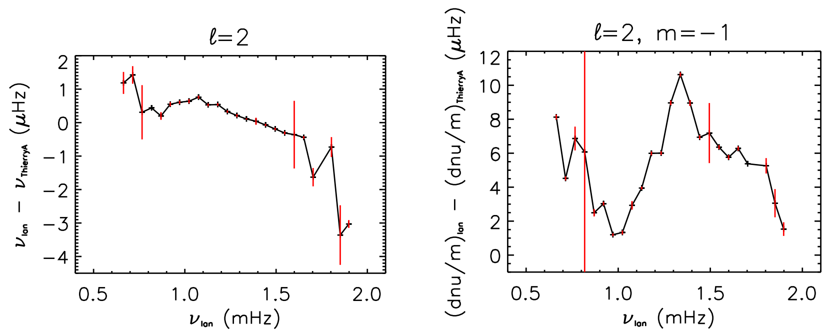

Figure 5. Comparison between the parameters fitted by the *Meudon* team with the theoretical parameters of the *Queen Mary* team; (left) for the $l = 2$ mode frequencies, (right) for the $l = 2, m = 1$ mode splitting. The agreement between the input and the fit is extremely poor. The misidentification of the $l = 2$ is obvious; the ridge identified as an $l = 2, m = 0$ was as a matter of fact a $l = 3, m = -2$.

5.3.3. *A difficult case: Queen Mary synthetic data*

Figure 3 shows the echelle diagramme for the *Queen Mary* synthetic data. The time series are 150-day long and sampled at 32 sec. The identification as explained in the caption is somewhat more difficult, to say the least. The main problem is that it seemed that the mode visibility was not according to what is usually expected, in addition the large 'apparent' rotational splitting confused even more the data fitters. The $l = 0$ and $l = 1$ modes were identified with some confidence. De facto we did not assume a common amplitude for the $l = 2$ modes constrained by the visibility of the multiplet as given by Toutain and Gouttebroze (1993). Instead we assumed that the $l = 2$ multiplet had

[8] The activity related to these HH exercise can be found in http://virgo.so.estec.esa.nl/html/corot/datagroup/hh.html

5 different amplitudes not constrained by geometrical visibility. This was as matter of fact a correct assumption verified *a posteriori*, but not sufficiently correct as it turned out that the $l = 3$ mode ridge was improperly identified as being $l = 2$.

Figures 4 and 5 shows the results obtained when comparing the output fitted parameters with those of the input theoretical parameters. The results obtained for Figure 4 are typical of what we obtained for other time series (See Berthomieu et al in these proceedings), or for the $l = 0$ of the *Queen-Mary* time series. Figure 5 is an example of what happens when modes are misidentified.

6. Conclusion

I have shown that mode identification and power spectrum fitting for solar-like stars benefit from 20 years of helioseismic experience. The methodology explained here can easily be applied when the signal-to-noise ratio is rather high. Hands-on experience for solar-like stars awaits the availability of space-based measurements (COROT, MOST). Within the COROT team, the waiting has been replaced by the use of hare-and-hound exercises. Examples of the results obtained within the COROT team show what can happen when one encounters the expected and the unexpected.

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